Raport științific final de evaluare a proiectului PN-III-P4-ID-PCE-2016-0011 „Analiza si controlul ecuației stochastice Schrödinger si a unor modele de difuzie neliniară” in perioada 12.07.2017-30.12.2019

Obiective:
Studiul problemei de control optimal atasata ecuației stochastice Schrodinger (existenta si principiul de maxim) si construirea unui control feedback pentru aceasta problema ca si pentru problema urmaririi orbitei pe liniile prezentate anterior. O teorie de existenta riguroasa pentru o problema stochastica parabolica si tehnici de restaurare a imaginilor bazate pe aceasta ecuatie cu neliniaritate nemonotona a similara celei din modelul Perona-Malik. Analiza si aproximarea unor modele matematice in biologie.

2017

In anul 2017, in cadrul proiectului PN-III-P4-ID-PCE-2016-0011 s-au realizat urmatoarele:

- 1 lucrare publicata in revista internationala ISI.
  

  Abstract. This paper is devoted to the study of noise effects on blow-up solutions to stochastic nonlinear Schrödinger equations. It is a continuation of our recent work [2], where the (local) well-posedness is established in $H^1$, also in the non-conservative critical case. Here we prove that in the non-conservative focusing mass-(super)critical case, by adding a large multiplicative Gaussian noise, with high probability one can prevent the blow-up on any given bounded time interval $[0,T]$, $0<T<\infty$. Moreover, in the case of spatially independent noise, the explosion even can be prevented with high probability on the whole time interval $[0,\infty)$. The noise effects obtained here are completely different from those in the conservative case studied in [5].

- 2 lucrari in curs de finalizare

  a. Gabriela Marinoschi, Analiza si aproximarea unor modele matematice in biologie bazate pe ecuatiile parabolice

  b. Tudor Barbu, O lucrare privind studiul problemei de restaurare a imaginilor folosind argumente stochastice.
- 5 conferinte:

- 1 workshop organizat:


2018

In anul 2018 in cadrul proiectului PN-III-P4-ID-PCE-2016-0011 s-au realizat urmatoarele:

- 10 lucrari publicate in reviste internationale ISI.


  **Abstract.** We show via the nonlinear semigroup theory in $L^1(R)$ that the 1-D dynamic programming equation associated with a stochastic optimal control problem with multiplicative noise has a unique mild solution in a sense to be made precise.


  **Abstract.** We analyze the bilinear optimal control problem of quantum mechanical systems with final observation governed by a stochastic nonlinear Schrödinger equation perturbed by a linear multiplicative Wiener process. The existence of an open-loop optimal control and first-order Lagrange optimality conditions are derived, via Skorohod's representation theorem, Ekeland's variational principle and the existence for the linearized dual backward stochastic equation. Moreover, our approach in particular applies to the deterministic case.

**Abstract.** One introduces a new variational concept of solution for the stochastic differential equation $dX + A(t)X dt + \dot{X}dt = X dW$, $t \in (0, T)$; $X(0) = x$ in a real Hilbert space where $A(t) = \bar{c}(t), t \in (0, T)$, is a maximal monotone subpotential operator in $H$ while $W$ is a Wiener process in $H$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. In this new context, the solution $X = X(t, x)$ exists for each $x \in H$, is unique, and depends continuously on $x$. This functional scheme applies to a general class of stochastic PDE so far not covered by the classical variational existence theory (Krylov and Rozovskii in J Sov Math 16:1233–1277, 1981; Liu and Röckner in Stochastic partial differential equations: an introduction, Springer, Berlin, 2015; Pardoux in Equations aux dérivées partielles stochastiques nonlinéaires monotones, Thèse, Orsay, 1972) and, in particular, to stochastic variational inequalities and parabolic stochastic equations with general monotone nonlinearities with low or superfast growth to $+\infty$.


**Abstract.** Existence of a strong solution in $H^1(\mathbb{R}^d)$ is proved for the stochastic nonlinear Fokker–Planck equation respectively, for a corresponding random differential equation. Here $d \geq 1$, $W$ is a Wiener process in $H^1(\mathbb{R}^d)$, $D\in C(\mathbb{R}^d, \mathbb{R}^d)$ and $\beta$ is a continuous monotonically increasing function satisfying some appropriate sublinear growth conditions which are compatible with the physical models arising in statistical mechanics. The solution exists for $x \in \mathbb{L}^{1/\epsilon} \cap \mathbb{L}^{\epsilon}$ and preserves positivity. If $\beta$ is locally Lipschitz, the solution is unique, pathwise Lipschitz continuous with respect to initial data in $H^1(\mathbb{R}^d)$. Stochastic Fokker–Planck equations with nonlinear drift of the form $dX = \text{div}(a(X))dt - \beta(X)dt = XdW$ are also considered for Lipschitzian continuous functions $a: \mathbb{R} \rightarrow \mathbb{R}^d$.


**Abstract.** In the present contribution we study the sliding mode control (SMC) problem for a diffuse interface tumor growth model coupling a viscous Cahn–Hilliard type equation for the phase variable with a reaction-diffusion equation for the nutrient. First, we prove the well-posedness and some regularity results for the state system modified by the state-feedback control law. Then, we show that the chosen SMC law forces the system to reach within finite time the sliding manifold (that we chose in order that the tumor phase remains constant in time). The feedback control law is added in the Cahn–Hilliard type equation and leads the phase onto a prescribed target $\varphi^*$ in finite time.


**Abstract.** A novel structure-based image interpolation technique is proposed in this paper. It is based on a nonlinear anisotropic diffusion model that is properly constructed for the reconstruction process. A rigorous mathematical investigation of this partial differential equation (PDE)-based scheme is then performed, its well-posedness being treated. An explicit finite difference-based numerical approximation scheme that is consistent to the second-order PDE model and converges to its weak solution is developed next. The successful inpainting experiments and method comparison prove the effectiveness of the considered diffusion-based approach.

Abstract. A novel second-order partial differential equation (PDE) - based image restoration technique is proposed here. The considered denoising method is based on a nonlinear hyperbolic differential model combined to a two-dimension filter kernel. The considered PDE model is well-posed and it is solved numerically by constructing an explicit iterative finite difference-based numerical approximation algorithm that is consistent to the combined PDE model and converges fast to its weak solution. Our successful restoration experiments and method comparison are also discussed.


Abstract. A hybrid nonlinear PDE-based denoising framework is proposed in this paper. The considered image restoration technique is based on a well-posed hyperbolic differential model that combines second- and fourth-order diffusions. A consistent finite difference-based numerical approximation algorithms is then constructed for solving this hyperbolic diffusion-based model. Our successful restoration experiments that illustrate the effectiveness of the proposed method are also discussed.


Abstract: In this work we design an explicit random deterministic, finite-dimensional stabilizing boundary feedback to the null solution for the heat equation with noise perturbation. The simple form of the feedback allows us to write the solution of the corresponding closed-loop equation in a mild formulation via a kernel and use some techniques from the existing literature in order to show the stability of it. As far as we know, the present work is the first result on boundary feedback stabilization for stochastic parabolic-type equations, with the stability guaranteed independent of how large the level of the noise is.


Abstract. In this work, we design explicit, finite-dimensional boundary feedback laws for stabilization to trajectories for parabolic-type equations. The simple form of the feedback allows to write the solution of the corresponding closed-loop equation in a mild formulation via a kernel; then, taking advantage of this, the stability is shown. As an application, null stabilization for stochastic parabolic-type equations is deduced as well. As far as we know, the present work is the first result on boundary feedback stabilization to trajectories and for stochastic parabolic-type equations, with stability guaranteed independent of how large the level of the noise is.

- 5 conferinte:


- s-a actualizat pagina web a proiectului:

2019

In anul 2019, in cadrul proiectului PN-III-P4-ID-PCE-2016-0011 s-au realizat urmatoarele:

5 lucrari publicate in reviste internationale ISI.


   Abstract. One proves that the \( n \)-D stochastic controlled equation \( dX(t) + A(t)X(t) \, dt = \sigma(X(t))dW(t) + B(t)u(t)dt \), where \( \sigma \in \text{Lip}(\mathbb{R}^n, L(\mathbb{R}^d, \mathbb{R}^n)) \), \( A(t) \in L(\mathbb{R}^n) \) and \( B(t) \in L(\mathbb{R}^n, \mathbb{R}^n) \) is invertible, is exactly controllable with high probability in each \( y \in \mathbb{R}^n \) such that \( \sigma(y) = 0 \) on each finite interval \((0, T)\). An application to approximate controllability of the stochastic heat equation is given. The case where \( B \in L(\mathbb{R}^m, \mathbb{R}^n), 1 \leq m < n \) and the pair \((A, B)\) satisfies the Kalman rank condition is also studied.


   Abstract. In this note, one constructs a distributional solution to the \( d \)-dimensional dynamic programming equation, \( d \geq 3 \), for an optimal control problem governed by a stochastic volatility model. The approach is based on nonlinear semigroup theory in the space \( L^1(\mathbb{R}^d) \).


   Abstract. We are concerned with a nonlinear nonautonomous model represented by an equation describing the dynamics of an age-structured population diffusing in a space habitat \( O \), governed by local Lipschitz vital factors and by a stochastic behavior of the demographic rates possibly representing emigration, immigration and fortuitous mortality. The model is completed by a random initial condition, a flux type boundary conditions on \( \partial O \) with a random jump in the population density and a nonlocal nonlinear boundary condition given at age zero. The stochastic influence is expressed by a linear multiplicative Gaussian noise perturbation in the equation. The main result proves that the stochastic model is well-posed, the solution being in the class of path-wise continuous functions and satisfying some particular regularities with respect to the age and
space. The approach is based on a rescaling transformation of the stochastic equation into a random deterministic time dependent hyperbolic-parabolic equation with local Lipschitz nonlinearities. The existence and uniqueness of a strong solution to the random deterministic equation is proved by combined semigroup, variational and approximation techniques. The information given by these results is transported back via the rescaling transformation towards the stochastic equation and enables the proof of its well-posedness.


Abstract. This work represents a first contribution on the problem of boundary stabilization for the phase field system of Cahn-Hilliard type, which models the phase separation in a binary mixture. The feedback controller we design here is with actuation only on the temperature flow of the system, on one part of the boundary only. Moreover, it is of proportional type, given in an explicit form, expressed only in terms of the eigenfunctions of the Laplace operator, being easy to manipulate from the computational point of view. Furthermore, it ensures that the closed loop nonlinear system exponentially reaches the prescribed stationary solution provided that the initial datum is close enough to it.


Abstract. In this work controlled systems of semilinear parabolic equations are considered. Only one control is acting in both equations and it is distributed in a subdomain. Local feedback stabilization is studied. The approach is based on approximate controllability for the linearized system and the use of an appropriate norm obtained from a Lyapunov equation. Applications to reaction-diffusion systems are discussed.

1 capitol de carte


Abstract. An overview of additive noise removal algorithms using secondder nonlinear partial differential equations (PDEs) is provided in this paper. The state of the art anisotropic diffusion models for image restoration are described first. Then, the second-order PDE-based denoising approaches using variational schemes are addressed. Our most important contributions in these image processing fields are also mentioned in this work.

9 lucrari elaborate/trimise/acceptate la publicare


Abstract. This work is concerned with the time optimal control problem for evolution equations in Hilbert spaces. The attention is focused on the maximum principle for the time optimal controllers having the dimension smaller that of the state system, in particular for minimal time sliding mode controllers, which is one of the novelties of this paper. We provide the characterization of the controllers by the optimality conditions determined for some general cases. The proofs rely on a set of hypotheses meant to cover a large class of applications. Examples of control problems governed by parabolic equations with potential and drift terms, porous media equation or reaction-diffusion systems with linear and nonlinear perturbations, describing real world processes, are presented at the end.

Abstract. Chemotherapy is a common treatment for advanced prostate cancer. The standard approach relies on cytotoxic drugs, which aim at inhibiting proliferation and promoting cell death. Advanced prostatic tumors are known to rely on angiogenesis, i.e., the growth of local microvasculature via chemical signaling produced by the tumor. Thus, several clinical studies have been investigating antiangiogenic therapy for advanced prostate cancer, either as monotherapy or in combination with standard cytotoxic protocols. However, the complex genetic alterations that originate and sustain prostate cancer growth complicate the selection of the best chemotherapeutic approach for each patient’s tumor. Here, we present a mathematical model of prostate cancer growth and chemotherapy that may enable physicians to test and design personalized chemotherapeutic protocols in silico. We use the phase-field method to describe tumor growth, which we assume to be driven by a generic nutrient following reaction-diffusion dynamics. Tumor proliferation and apoptosis (i.e., programmed cell death) can be parameterized with experimentally-determined values. Cytotoxic chemotherapy is included as a term downregulating tumor net proliferation, while antiangiogenic therapy is modeled as a reduction in intratumoral nutrient supply. An additional equation couples the tumor phase field with the production of prostate-specific antigen, which is a prostate cancer biomarker that is extensively used in the clinical management of the disease. We prove the well-posedness of our model and we run a series of representative simulations leveraging an isogeometric method to explore untreated tumor growth as well as the effects of cytotoxic chemotherapy and antiangiogenic therapy, both alone and combined. Our simulations show that our model captures the growth morphologies of prostate cancer as well as common outcomes of cytotoxic and antiangiogenic mono and combined therapy. Additionally, our model also reproduces the usual temporal trends in tumor volume and prostate-specific antigen evolution observed in experimental and clinical studies.


Abstract. A novel fourth-order partial differential equation (PDE) – based image restoration technique is proposed in this work. It is based on a well-posed fourth-order nonlinear diffusion-based model combined to a two-dimension filter kernel. An iterative finite difference-based numerical approximation algorithm is then constructed for solving the PDE model. The proposed approach removes successfully the additive noise, overcome unintended effects like the staircasing and preserves successfully the edges and other image details.


Abstract: A partial differential equation (PDE) – based technique for filtering the Poisson noise from digital images is proposed in this work. It is based on a nonlinear second-order anisotropic diffusion-based model that is adapted for the Poisson distribution. The considered PDE model is well-posed and its unique and weak solution is computed using a finite difference-based numerical approximation scheme that is consistent to the proposed model. The proposed approach provides an effective feature-preserving Poisson denoising. Some results of our filtering simulations are also described in this paper.
5. Catalin-George Lefter, Elena-Alexandra Melnig, On the parabolic regularity, Sobolev embeddings and global Carleman estimates in $L^q(U)$ spaces, Pure and Applied Functional Analysis, acceptata. ISSN 2189-3756.

Abstract. In this paper we discuss some aspects related to regularity in parabolic problems with corollaries regarding anisotropic Sobolev embeddings. We use these results in the context of bootstrap arguments applied to global Carleman estimates for nonhomogeneous parabolic equations in $L^q(U)$ spaces, estimates which are fundamental in associated control and inverse problems. The arguments we use are characterizations of regularity in terms of domains of fractional powers of elliptic operators and then characterization of these domains as interpolation spaces and relations to Bessel potential and Sobolev-Slobodeckii spaces.


Abstract. One proves the $H$-theorem for mild solutions to a nondegenerate, nonlinear Fokker-Planck equation

$$u_t - \Delta \beta(u) + \text{div}(D(x)b(u)u) = 0, \quad t \geq 0, \quad x \rightarrow \mathbb{R}^d,$$

and under appropriate hypotheses on $\beta$, $D$, and $b$ the convergence in $L^1_{\text{loc}}(\mathbb{R}^d)$, $L^1(\mathbb{R}^d)$, respectively, for some $t_n \rightarrow \infty$ of the solution $u(t_n)$ to an equilibrium state of the equation for a large set of nonnegative initial data in $L^1$. Furthermore, the solution to the McKean-Vlasov stochastic differential equation corresponding to (1), which is a nonlinear distorted Brownian motion, is shown to have this equilibrium state as its unique invariant measure.

7. Viorel Barbu, Michael Röckner, From nonlinear Fokker-Planck equations to solutions of distribution dependent SDE.

Abstract. We construct weak solutions to the McKean-Vlasov SDE

$$dX(t) = b(X(t),(dLX(t)/dx)(X(t)))dt + \sigma(X(t),(dLX(t)/dx)(X(t)))dW(t)$$

on $\mathbb{R}^d$ for possibly degenerate diffusion matrices $\sigma$ with $X(0)$ having a given law, which has a density with respect to Lebesgue measure, $dx$. Here $LX(t)$ denotes the law of $X(t)$. Our approach is to first solve the corresponding nonlinear Fokker-Planck equations and then use the well known superposition principle to obtain weak solutions of the above SDE.

8. Elena-Alexandra Melnig, Stability in $L^q$-norm for inverse source parabolic problems, submitted to Journal of Inverse and Ill Posed Problems. (ISI)

Abstract. We consider systems of parabolic equations coupled in zero and first order terms. We establish Lipschitz estimates in $L^q$-norms, $2 \leq q \leq \infty$ for the source in terms of the solution in a subdomain. The main tool is a family of appropriate Carleman estimates with general weights, in Lebesgue spaces, for nonhomogeneous parabolic systems.


Abstract. We consider coupled parabolic systems with homogeneous boundary conditions. We establish a family of $L^q$-Carleman inequalities, $q \in [2, \infty)$ and use them to obtain stability estimates in $L^q$ and $L^r$ norms for the sources in terms of the solution in a subdomain. We apply these estimates to reaction-diffusion systems.
6 Conferințe


3 Stagii de cercetare

1. Viorel Barbu, Stagiul de cercetare cu grupul “Taming uncertainty and profiting from randomness and low regularity in analysis, stochastic and their applications” din cadrul Universitatii din Bielefeld, în perioada 05 august – 05 septembrie 2019.


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[Signature]

Acad. Viorel Barbu